

Homework 5

2.

a) Annual Stellar Parallax is the apparent movement of a nearby star with respect to the more distant background stars. Maximum apparent displacement comes at roughly 6 month intervals, when the Earth is on opposite sides of the sun. In a sense, this parallax is an optical illusion—the stars appear to move, but it is the Earth which is really moving.

b) This should be some sort of diagram indicating that parallax is best measured from the Earth six months apart, in order to give the longest baseline (1AU) for the measurement. This diagram should look very similar to the diagram on page 7 of Dr. Kolb's lecture 8 (April 27).

c) You may certainly use the equation $\tan(\alpha) = \frac{1AU}{D}$, but this is more difficult than it needs to be. If you do use this equation, remember there is no need to convert your angle from degrees to radians. Provided your calculator knows whether it is getting degrees or radians, either may be entered in your calculator.

For small angles (and given that the closest star to the sun is at $\sim 1pc$ and, therefore, has a parallactic angle of 1 arc second, all parallactic angles are small), you may use the simpler formula $D = \frac{1}{\alpha}$. Remember that, in this formula, distances will be in parsecs if the angle is in arc seconds. This problem just becomes:

$$D = \frac{1}{0.02} = 50pc.$$

3.

Using $\pi = 3.14$;

Angle in degrees	Angle in radians	Tangent of angle	Sine of angle
1°	0.0175	0.0175	0.0175
3°	0.0524	0.0524	0.0524
10°	0.1745	0.1763	0.1736
30°	0.5236	0.5774	0.5
50°	0.8727	1.1918	0.7660

4.

Important formulae to keep in mind when solving problems like this one:

$$L = \frac{I}{4\pi R^2} \quad (1)$$

where L is the luminosity of the star, I the measured intensity of the star and R the distance to it.

$$m_1 - m_2 = 2.5 \log \left(\frac{I_2}{I_1} \right) = -2.5 \log \left(\frac{I_1}{I_2} \right) \quad (2)$$

Now, let's proceed to the solution to the problem.

- a) In our case we know the luminosity of W-Bush which is 0.04 times the Solar Luminosity. $L_{WB} = 0.04 \cdot L_{\odot}$, the distance to it $R_A = 10 \text{ pc}$ and its apparent magnitude is unknown. We also know the distance to the sun and its magnitude $m_{\odot} = -26.8$. Now, lets apply equation 2:

$$\begin{aligned}
 m_{WB} - m_{\odot} &= -2.5 \log \left(\frac{I_{WB}}{I_{\odot}} \right) \Rightarrow \\
 m_{WB} - (-26.8) &= -2.5 \log \left(\frac{I_{WB}}{I_{\odot}} \right) \Rightarrow \\
 m_{WB} &= -26.8 - 2.5 \log \left(\frac{I_{WB}}{I_{\odot}} \right) \quad (3)
 \end{aligned}$$

Lets keep this in mind and move on to using equation 1:

$$\begin{aligned}
 \frac{I_{WB}}{I_{\odot}} &= \frac{\frac{L_{WB}}{4\pi R_{WB}^2}}{\frac{L_{\odot}}{4\pi R_{\odot}^2}} \\
 &= \frac{L_{WB} R_{\odot}^2}{L_{\odot} R_{WB}^2} \\
 &= \frac{0.04 \cdot L_{\odot} (1AU)^2}{L_{\odot} \cdot (10pc)^2} \\
 &= \frac{0.04 \cdot (1AU)^2}{(200000)^2 (AU)^2} \\
 &= \frac{4 \cdot 10^{-2}}{4 \cdot 10^{10}} \\
 \frac{I_{WB}}{I_{\odot}} &= 10^{-12} \quad (4)
 \end{aligned}$$

But, the ratio of the intensities is what we were looking for. Therefore, plugging in equation 4 we get:

$$\begin{aligned}
 m_{WB} &= -26.8 - 2.5 \log(10^{-12}) = -26.8 - 2.5 \cdot (-12) = -26.8 + 30 \Rightarrow \\
 m_{WB} &= 3.2
 \end{aligned}$$

- b) We know the distance in parsecs and we want to find the parallax:

$$\alpha_{WB}["] = \frac{1}{D[pc]} = \frac{1}{10} \Rightarrow \alpha_{WB} = 1''$$

5.

a) We want the apparent brightness to be the same. Remember that the brightness is given by:

$$b = \frac{L}{4\pi r^2} \quad (5)$$

Therefore

$$\frac{4 \times 10^{28} W}{4\pi(10 pc)^2} = \frac{40 W}{4\pi(r)^2} \quad (6)$$

Solving for r we get

$$r = 3.16 \times 10^{-13} pc \quad (7)$$

b) Here we need to transform our previous result from pc to cm

$$r = 3.16 \times 10^{-13} pc \times \frac{4 \times 10^{18} cm}{1 pc} = 1.26 \times 10^6 cm \quad (8)$$